4.16*a*

Solving Logarithmic Equations

One-to-one properties:

For any exponential function $f(x) = b^x$,

- If $b^u = b^v$, then $u = v$.

For any logarithmic function $f(x) = \log_b x$,

- If $\log_b u = \log_b v$, then $u = v$.

Solving Exponential Equations

Example:

Solve: $20\left(\frac{1}{2}\right)^{\frac{x}{3}} = 5$

$20\left(\frac{1}{2}\right)^{\frac{x}{3}} = 5$ $\quad \div \text{both sides by } 20$

$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \frac{1}{4}$ $\quad$ *rewrite with same base*

$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \left(\frac{1}{2}\right)^{2}$ $\quad$ *one – to – one property*

$\frac{x}{3} = 2$

$x = 6$

Solving Logarithmic Equations

Example:

Method 1:

Solve $\log x^2 = 2$.

$\log x^2 = 2$ *(rewrite in exponential form)*

$10^2 = x^2$ *(solve for x)*

x = 10 or -10

Method 2:

Solve $\log x^2 = 2$.

$\log x^2 = \log 10^2 \qquad (y = \log 10^x)$

$x^2 = 10^2 \qquad (one-to-one\ property)$

$x = 100$

$x = 10 \quad or \quad x = -10$

Method 3 (Incorrect!):

Solve $\log x^2 = 2$.

$\log x^2 = 2$

$2 \log x = 2 \qquad (\text{power rule, applied incorrectly})$

$\log x = 1 \qquad (\div \text{ by } 2)$

$x = 10 \qquad (\text{change to exp. form})$

Solving a Logarithmic Equation:

Solve ln (3x - 2) + ln (x - 1) = 2ln x.

ln (3x - 2) + ln (x - 1) = 2ln x

ln [(3x - 2)(x - 1)] = ln x^2

(3x - 2)(x - 1) = x^2

$3x^2 - 3x - 2x + 2 = x^2$

$2x^2 - 5x + 2 = 0$

(2x - 1)(x - 2) = 0

x = ½ or x = 2 check domain $x \neq ½$ so x = 2 is the only solution.

We can also solve this by graphing, setting the original equation equal to 0 and finding the x-intercepts.

Solve ln (3x - 2) + ln (x - 1) = 2ln x.

ln (3x - 2) + ln (x - 1) - 2ln x = 0

X intercept is at (2, 0).

Domain of a logarithm: See pgs. 173-174.

See more example of solving in book pgs. 174- 179.

4.16b

Finding the Inverse of a Logarithmic or Exponential Function:

(See pg. 176 in book)

Using the Structure of Expressions to Solve Equations (Honors)

(See pg. 179 in book)